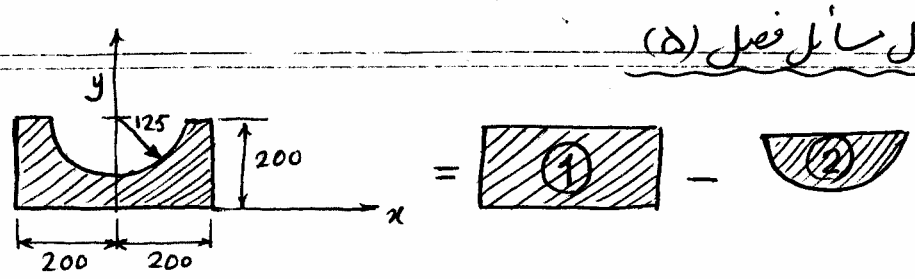
5.8



C | $\bar{x}$ / $\bar{y}$

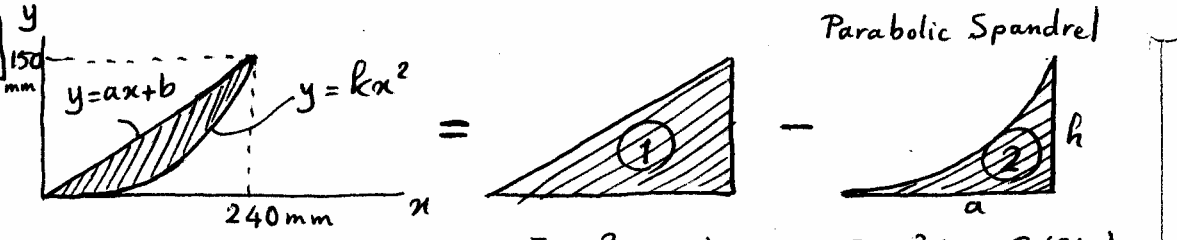
$$\bar{x} = 0$$

$$\bar{y} = \frac{\Sigma \bar{y}_i A_i}{\Sigma A_i} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2}{A_1 + A_2} =$$

$$= \frac{100(200\times400) + \left(200 - \frac{4\times125}{3\pi}\right)\left(-\frac{1}{2}\pi 125^2\right)}{200\times400 + \left(-\frac{1}{2}\pi\times125^2\right)}$$

$$= 79.22 \text{ mm}$$

5.12



C | $\bar{X}$ / $\bar{Y}$

$$\bar{x}_1 = \frac{2}{3}(240), \quad \bar{x}_2 = \frac{3a}{4} = \frac{3(240)}{4}$$

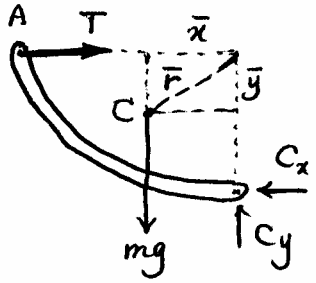
$$\bar{y}_1 = \frac{1}{3}(150), \quad \bar{y}_2 = \frac{3h}{10} = \frac{3(150)}{10}$$

$$A_1 = \frac{1}{2}(240)(150), \quad A_2 = -\frac{ah}{3} = -\frac{240\times150}{3}$$

$$\bar{X} = \frac{\Sigma \bar{x}_i A_i}{\Sigma A_i} = \frac{\frac{2}{3}(240)\times\frac{1}{2}240\times150 + \frac{3}{4}240\left(-\frac{240\times150}{3}\right)}{\frac{1}{2}240\times150 - \frac{240\times150}{3}} = 120 \text{ mm}$$

$$\bar{Y} = \frac{\Sigma \bar{y}_i A_i}{\Sigma A_i} = \frac{\frac{1}{3}(150)A_1 + 3\times150/10\, A_2}{A_1 + A_2} = 60 \text{ mm}$$

5.30



$$\bar{x} = \bar{y} = \frac{2r}{\pi}$$

$$\Sigma M_C = 0 \Rightarrow mg\bar{x} - Tr = 0$$

$$\Rightarrow T = \frac{mg\bar{x}}{r} = \frac{4(9.81)\left(\frac{2\times250}{\pi}\right)}{250}$$

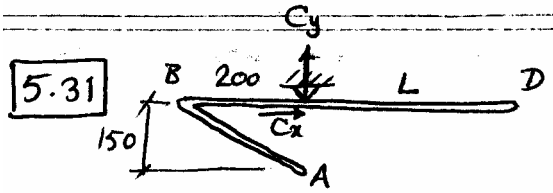
$$= 24.98 \text{ N}$$

$$\Sigma F_x = 0 \Rightarrow -C_x + T = 0 \Rightarrow C_x = T = 24.98\text{N}$$

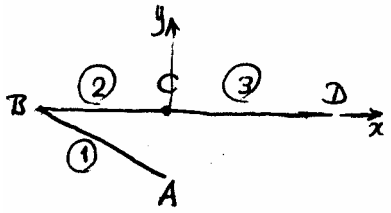
$$\Sigma F_y = 0 \Rightarrow C_y - mg = 0 \Rightarrow C_y = mg = 39.24\text{N}$$

$$\Rightarrow C = \sqrt{C_x^2 + C_y^2} = 46.52\text{N}, \angle 57.5^\circ$$

5.31



برای آنکه در حالت تعادل، قسمت BCD افقی باشد، باید در حالت نشان داده شده، مرکز جرم کل درست در زیر نقطهٔ C قرار گیرد، یعنی اگر نقطهٔ C مبدأ مختصات باشد، باید $\bar{x}$ مرکز جرم صفر باشد.

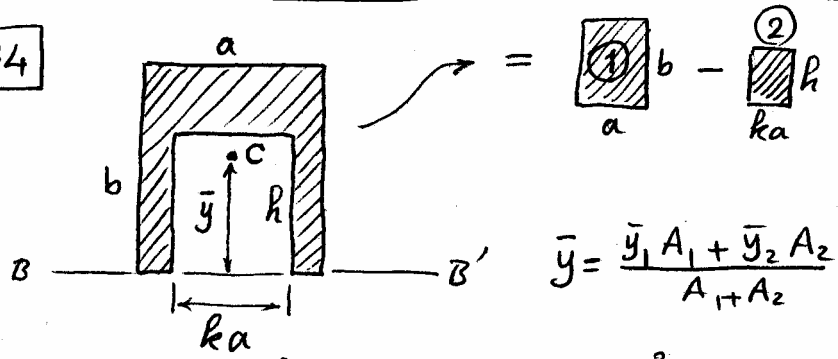


$$\bar{X} = \frac{\sum \bar{x}_i L_i}{\sum L_i} = \frac{\bar{x}_1 L_1 + \bar{x}_2 L_2 + \bar{x}_3 L_3}{L_1 + L_2 + L_3} = 0$$

$$\Rightarrow \frac{(100)\sqrt{150^2+200^2} + (-100)(200) + (\frac{L}{2})L}{\sqrt{150^2+200^2} + 200 + L} = 0$$

$$\Rightarrow \frac{L^2}{2} = 45000 \Rightarrow \boxed{L = 300\ mm}$$

5.34



$$\bar{y} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2}{A_1 + A_2}$$

$$\Rightarrow \bar{y} = \frac{\frac{b}{2}(ab) + (\frac{h}{2})(-kah)}{ab - kah} = \frac{b^2 - kh^2}{2(b-kh)}$$

$$\frac{d\bar{y}}{dh} = 0 \Rightarrow \frac{-2kh(2(b-kh)) + 2k(b^2 - kh^2)}{4(b-kh)^2} = 0$$

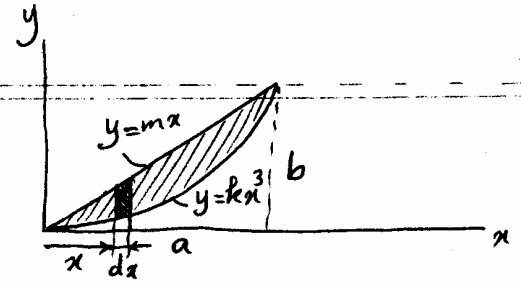
$$\Rightarrow -4khb + 4k^2h^2 + 2kb^2 - 2k^2h^2 = 0 \Rightarrow k^2h^2 - 2khb + kb^2 = 0$$

$$\Rightarrow kh^2 - 2bh + b^2 = 0 \Rightarrow h = \frac{+b \pm \sqrt{b^2 - kb^2}}{k} = \frac{b(1 \pm \sqrt{1-k})}{k}$$

الف) $k = 0.1 \longleftarrow h = 0.513b$

ب) $k = 0.8 \longleftarrow h = 0.691b$

5.42



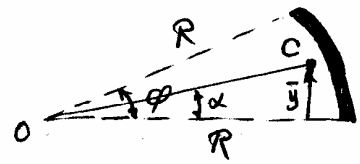
$x=a \rightarrow y=b \rightarrow \begin{cases} b=ma \rightarrow m=\frac{b}{a} \\ b=ka^3 \rightarrow k=\frac{b}{a^3} \end{cases}$

$$A=\int_0^a dA=\int_0^a (mx-kx^3)dx=\left(\frac{mx^2}{2}-\frac{kx^4}{4}\right)\Big|_0^a=\frac{ma^2}{2}-\frac{ka^4}{4}$$
$$=\frac{1}{2}\frac{b}{a}a^2-\frac{1}{4}\frac{b}{a^3}a^4=\frac{ab}{2}-\frac{ab}{4}=\boxed{\frac{ab}{4}}$$

$$A\bar{x}=\int \bar{x}_{el}\,dA \Rightarrow \bar{x}A=\int_0^a x(mx-kx^3)dx=\frac{ma^3}{3}-\frac{ka^5}{5}$$
$$=\frac{a^2b}{3}-\frac{a^2b}{5}=\frac{2a^2b}{15} \Rightarrow \boxed{\bar{x}=\frac{2a^2b/15}{ab/4}=\frac{8a}{15}}$$

$$\bar{y}A=\int \bar{y}_{el}\,dA \Rightarrow \bar{y}A=\int_0^a \frac{mx+kx^3}{2}(mx-kx^3)dx=\int_0^a \frac{m^2x^2-k^2x^6}{2}dx$$
$$=\frac{m^2a^3}{6}-\frac{k^2a^7}{14}=\frac{1}{6}\frac{b^2}{a^2}a^3-\frac{1}{14}\frac{b^2}{a^6}a^7=\frac{ab^2}{6}-\frac{ab^2}{14}=\frac{4ab^2}{42}=\frac{2ab^2}{21}$$
$$\Rightarrow \boxed{\bar{y}=\frac{2ab^2/21}{ab/4}=\frac{8b}{21}}$$

5.60



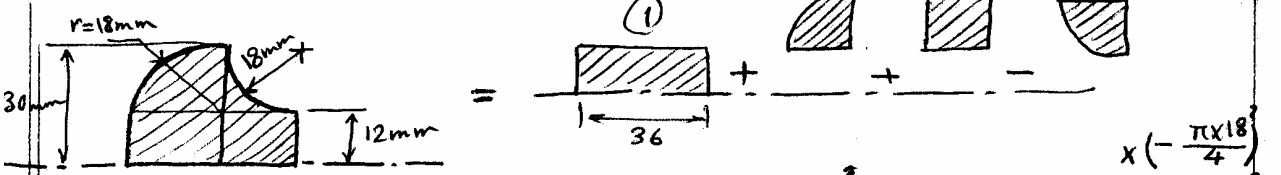
از جدول $\overline{OC}=\frac{R\sin\alpha}{\alpha}$ $\quad \alpha=\frac{\varphi}{2}=30°$

$\bar{y}=\overline{OC}\sin\alpha=\frac{R\sin^2\alpha}{\alpha}$

قضیه اول پاپوس $A=(2\pi\bar{y})L=\left(2\pi\frac{R\sin^2\alpha}{\alpha}\right)(R\varphi)$

$$=4\pi R^2\sin^2\alpha=4\pi(250^2)\sin^2 30°$$
$$=\boxed{196349.5\ mm^2}$$

5.66 $\rho=8470\ kg/m^3$

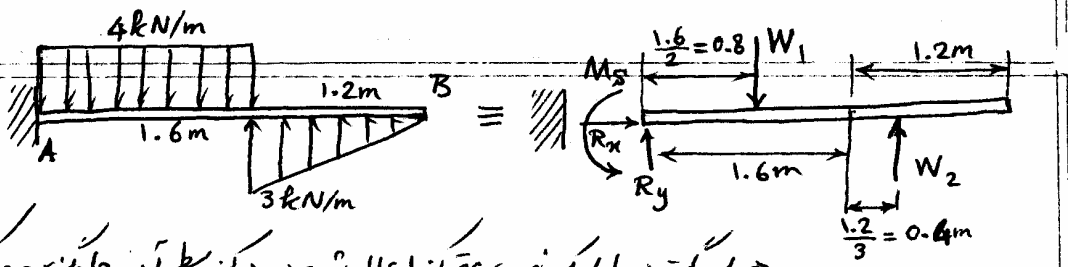


$$\bar{Y}=\frac{\Sigma \bar{y}_i A_i}{\Sigma A_i}=\frac{6(12\times36)+\left(\frac{4\times18}{3\pi}+12\right)\left(\frac{\pi\times18^2}{4}\right)+(12+9)(18\times18)+\left(30-\frac{4\times18}{3\pi}\right)\left(-\frac{\pi\times18^2}{4}\right)}{12\times36+\frac{\pi\times18^2}{4}+18\times18-\frac{\pi\times18^2}{4}}$$
$$=\boxed{11.513\ mm}$$

قضیه دوم پاپوس $V=(2\pi\bar{Y})A=2\pi\times11.513\times(12\times36+18\times18)$

$$=\boxed{54686.1\ mm^3} \qquad \boxed{m=\rho V=0.463\ kg}$$

5.76



هر بار گسترده را با یک نیروی متمرکز اعمال شده در مرکز سطح آن جایگزین می‌کنیم. مقدار هر نیروی متمرکز برابر با مساحت زیر نمودار می‌باشد.

$W_1 = 1.6 \times 4 \times 10^3 = 6.4\,kN$

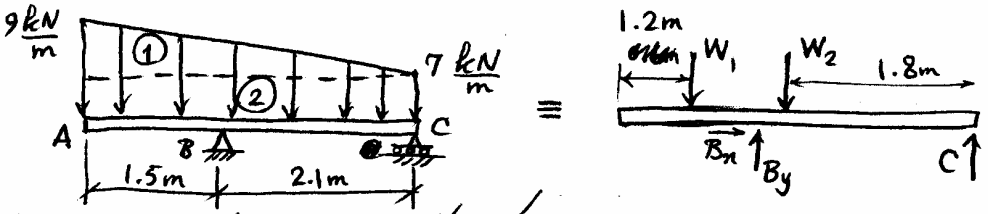
$W_2 = \frac{1}{2}(1.2) \times 3 \times 10^3 = 1.8\,kN$

$\Sigma F_x = 0 \Rightarrow \underline{R_x = 0}$ $\quad \Sigma F_y = 0 \Rightarrow R_y - W_1 + W_2 = 0 \Rightarrow \underline{R_y = W_1 - W_2 = 4.6\,kN}$

$\Sigma M_A = 0 \Rightarrow W_2(1.6+0.4) - W_1(0.8) + M_S = 0$

$\Rightarrow M_S = 6.4 \times 0.8 - 1.8(1.6+0.4) = \underline{1.52\,kN.m}$

5.81



فرض می‌کنیم بارگذاری ذوزنقه‌ای شکل از دو قسمت مستطیلی و مثلثی تشکیل شده است.

$W_1 = \frac{1}{2}(9-7)(3.6) = 3.6\,kN$, $W_2 = 7 \times 3.6 = 25.2\,kN$

$\Sigma F_x = 0 \Rightarrow \underline{B_x = 0}$, $\Sigma M_C = 0 \Rightarrow W_2(1.8) + W_1(3.6-1.2) - B_y(2.1) = 0$

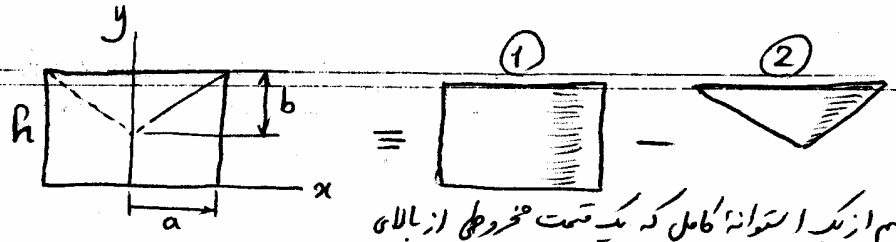
$\Rightarrow \underline{B_y = 25.71\,kN}$

$\Sigma F_y = 0 \Rightarrow B_y + C - W_1 - W_2 = 0 \Rightarrow \underline{C = W_1 + W_2 - B_y = 3.09\,kN}$

5.93

(سطوح غوطه‌ور حذف)

5.102



فرض می‌کنیم جسم از یک استوانهٔ کامل که یک قسمت مخروطی از بالای آن حذف شده، تشکیل شده‌است. از تقارن جسم معلوم است که: $\bar{x}=\bar{z}=0$

$$\bar{Y}=\frac{\sum \bar{y}_i V_i}{\sum V_i}=\frac{\bar{y}_1 V_1+\bar{y}_2 V_2}{V_1+V_2}$$

$$\bar{y}_1=\frac{h}{2},\quad V_1=\pi a^2 h$$

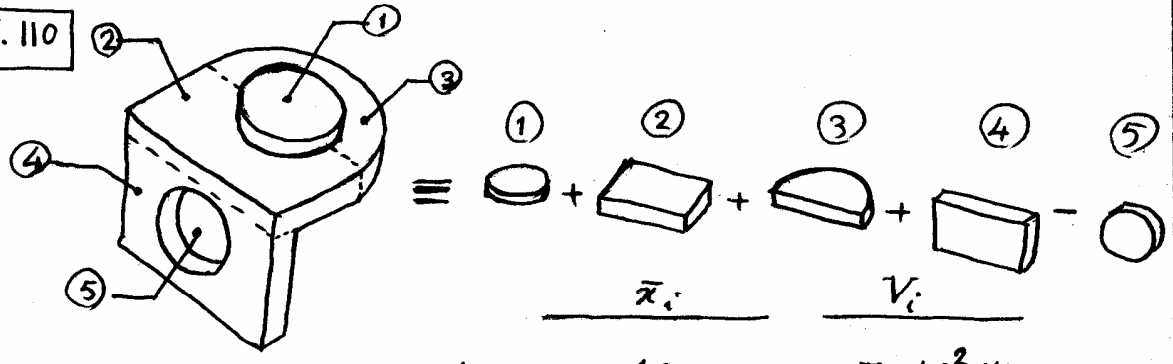
از جدول $\rightarrow$ $\bar{y}_2=h-\frac{b}{4}$, $V_2=\frac{1}{3}\pi a^2 b$

$$\rightarrow \bar{Y}=\frac{\frac{h}{2}(\pi a^2 h)+(h-\frac{b}{4})(-\frac{1}{3}\pi a^2 b)}{\pi a^2 h-\frac{1}{3}\pi a^2 b}=\frac{h^2/2+(h-b/4)(-b/3)}{h-b/3}$$

الف) $b=\frac{h}{3}$ $\rightarrow$ $\bar{Y}=\frac{h^2/2-h/9\,(h-h/12)}{h-h/9}=0.448h$

ب) $b=\frac{h}{2}$ $\rightarrow$ $\bar{Y}=\frac{h^2/2-h/6\,(h-h/8)}{h-h/6}=0.425h$

5.110



$$\bar{X}=\frac{\sum \bar{x}_i V_i}{\sum V_i}$$

	$\bar{x}_i$	V_i
1:	60	$\pi\times 40^2\times 10$
2:	$\frac{60}{2}=30$	$60\times 120\times 10$
3:	$60+\frac{4\times 60}{3\pi}$	$\frac{1}{2}\pi\times 60^2\times 10$
4:	$\frac{10}{2}=5$	$90\times 120\times 10$
5:	$\frac{10}{2}=5$	$-\pi\times 30^2\times 10$

$\rightarrow$ $\bar{X}=40.255\ mm$

5.134

(روش انتگرال‌گیری سه‌بعدی حذف)